EXAM CONCEPT BUILDER

IGCSE Mathematics - Chapter 1 - David Rayner 6th Edition

Exercise 1.1D

Fractions and Decimals - Full Step-by-Step Solutions

Questions 1 to 24 - Change the Fractions to Decimals

TIP: To convert a fraction to a decimal: divide the numerator (top) by the denominator (bottom). Use long division. If digits repeat, write with a dot above the repeating digit(s).

Type	What it means	Example
Terminating	The division ends exactly	1/4 = 0.25 (ends)
Recurring	One or more digits repeat forever	1/3 = 0.333... = 0.3 (dot above 3)

1. 1/4 **0.25** *= 0.25*	**2.** 2/5 **0.4** *= 0.4*	**3.** 4/5 **0.8** *= 0.8*	**4.** 3/4 **0.75** *= 0.75*
5. 1/2 **0.5** *= 0.5*	**6.** 3/8 **0.375** *= 0.375*	**7.** 9/10 **0.9** *= 0.9*	**8.** 5/8 **0.625** *= 0.625*
9. 5/12 **0.416 recurring** *Divide 5 by 12: recurring*	**10.** 1/6 **0.16 recurring** *Divide 1 by 6: recurring*	**11.** 2/3 **0.6 recurring** *Divide 2 by 3: recurring*	**12.** 5/6 **0.83 recurring** *Divide 5 by 6: recurring*
13. 2/7 **0.285714 recurring** *Divide 2 by 7: recurring*	**14.** 3/7 **0.428571 recurring** *Divide 3 by 7: recurring*	**15.** 4/9 **0.4 recurring** *Divide 4 by 9: recurring*	**16.** 5/11 **0.45 recurring** *Divide 5 by 11: recurring*
17. 1 and 1/5 **1.2** *= 1.2*	**18.** 2 and 5/8 **2.625** *= 2.625*	**19.** 2 and 1/3 **2.3 recurring** *Divide 1 by 3: recurring*	**20.** 1 and 7/10 **1.7** *= 1.7*
21. 2 and 3/16 **2.1875** *= 2.1875*	**22.** 2 and 2/7 **2.285714 recurring** *Divide 2 by 7: recurring*	**23.** 2 and 6/7 **2.857142 recurring** *Divide 6 by 7: recurring*	**24.** 3 and 19/100 **3.19** *= 3.19*

Worked Example - Q1: 1/4 to decimal

1 DIVIDE 1 BY 4

$\frac{1}{4}$ means 1 / 4

2 **LONG DIVISION**

1.00 / 4: 4 into 10 = 2 rem 2, 4 into 20 = 5 rem 0

3 **RESULT**

0.25 (terminates exactly)

ANSWER: 0.25

Worked Example - Q10: 1/6 to decimal (recurring)

1 **DIVIDE 1 BY 6**

$\frac{1}{6}$ means 1 / 6

2 **LONG DIVISION**

1.000 / 6: 6 into 10 = 1 rem 4, 6 into 40 = 6 rem 4, 6 into 40 = 6 rem 4 ...

3 **PATTERN SPOTTED**

The remainder 4 keeps repeating -> the digit 6 recurs

4 **WRITE ANSWER**

0.1666... = 0.16 (dot above the 6 to show it recurs)

ANSWER: 0.16 recurring (dot above 6)

Worked Example - Q13: 2/7 to decimal (recurring)

1 **DIVIDE 2 BY 7**

2.0000000 / 7

2 **LONG DIVISION**

7 into 20=2 r6, 7 into 60=8 r4, 7 into 40=5 r5, 7 into 50=7 r1, 7 into 10=1 r3, 7 into 30=4 r2, 7 into 20=2 r6 (back to start)

3 **6 DIGITS RECUR**

0.285714285714... = 0.285714 (dots above 2 and 4)

ANSWER: 0.285714 recurring (dots above first and last digit)

Worked Example - Q17: 1 and 1/5 to decimal

1 **KEEP THE WHOLE NUMBER PART**

Whole number = 1

2 **CONVERT THE FRACTION PART**

$\frac{1}{5}$ = 1 / 5 = 0.2

3 **ADD TOGETHER**

1 + 0.2 = 1.2

ANSWER: 1.2

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Questions 25 to 40 - Change Decimals to Fractions and Simplify

TIP: To convert a decimal to a fraction: count the decimal places. 1 d.p. = over 10, 2 d.p. = over 100, 3 d.p. = over 1000. Then simplify by dividing top and bottom by the GCF.

25. 0.2 **1/5** *2/10 / GCF=2*	**26.** 0.7 **7/10** *7/10 / GCF=1*	**27.** 0.25 **1/4** *25/100 / GCF=25*	**28.** 0.45 **9/20** *45/100 / GCF=5*
29. 0.36 **9/25** *36/100 / GCF=4*	**30.** 0.52 **13/25** *52/100 / GCF=4*	**31.** 0.125 **1/8** *125/1000 / GCF=125*	**32.** 0.625 **5/8** *625/1000 / GCF=125*
33. 0.84 **21/25** *84/100 / GCF=4*	**34.** 2.35 **2 and 7/20** *235/100 / GCF=5*	**35.** 3.95 **3 and 19/20** *395/100 / GCF=5*	**36.** 1.05 **1 and 1/20** *105/100 / GCF=5*
37. 3.2 **3 and 1/5** *32/10 / GCF=2*	**38.** 0.27 **27/100** *27/100 / GCF=1*	**39.** 0.007 **7/1000** *7/1000 / GCF=1*	**40.** 0.00011 **11/100000** *11/100000 / GCF=1*

Worked Example - Q27: 0.25 to fraction

1 COUNT DECIMAL PLACES

0.25 has 2 decimal places -> write over 100

2 WRITE AS FRACTION

$$\frac{25}{100}$$

3 FIND GCF OF 25 AND 100

GCF = 25

25 = 5x5, 100 = 4x25

4 SIMPLIFY

$$\frac{25}{100} = \frac{25:25}{100:25} = \frac{1}{4}$$

ANSWER: $\frac{1}{4}$

Worked Example - Q34: 2.35 to fraction

1 SEPARATE WHOLE NUMBER AND DECIMAL

$2.35 = 2 + 0.35$

2 CONVERT DECIMAL PART: 0.35 HAS 2 D.P.

$$\frac{35}{100}$$

3 SIMPLIFY: GCF(35,100) = 5

$$\frac{35}{100} = \frac{7}{20}$$

35/5=7, 100/5=20

4 ADD BACK WHOLE NUMBER

$$2 + \frac{7}{20} = 2\ \frac{7}{20}$$

ANSWER: $2\ \frac{7}{20}$

Worked Example - Q39: 0.007 to fraction

1 COUNT DECIMAL PLACES

0.007 has 3 decimal places -> write over 1000

2 WRITE AS FRACTION

$$\frac{7}{1000}$$

3 CHECK GCF

GCF(7,1000) = 1 -> already simplest form

7 is prime

ANSWER: $\frac{7}{1000}$

Questions 41 to 48 - Evaluate to 2 Decimal Places

TIP: Calculate the exact answer first, then round to 2 decimal places. Look at the 3rd decimal place: if it is 5 or more, round up; if less than 5, round down.

41.	42.	43.	44.
1/4 + 1/3	2/3 + 0.75	8/9 - 0.24	7/8 + 5/9 + 2/11
0.58	**1.42**	**0.65**	**1.61**
exact = 0.583333	*exact = 1.416667*	*exact = 0.648889*	*exact = 1.612374*

45.	46.	47.	48.
1/3 x 0.2	5/8 x 1/4	8/11 / 0.2	(4/7 - 1/3) / 0.4
0.07	**0.16**	**3.64**	**0.60**
exact = 0.066667	*exact = 0.156250*	*exact = 3.636364*	*exact = 0.595238*

Worked Example - Q41: 1/4 + 1/3 (to 2 d.p.)

1 CONVERT TO DECIMALS

$\frac{1}{4}$ = 0.25 $\frac{1}{3}$ = 0.3333...

2 ADD

0.25 + 0.3333... = 0.5833...

3 ROUND TO 2 D.P.

0.5833... -> 3rd d.p. is 3 (less than 5) -> round down

ANSWER: 0.58

Worked Example - Q48: (4/7 - 1/3) / 0.4 (to 2 d.p.)

1 BIDMAS: BRACKET FIRST

$\frac{4}{7}$ - $\frac{1}{3}$

2 LCM(7,3) = 21

$\frac{4}{7}$ = $\frac{12}{21}$ $\frac{1}{3}$ = $\frac{7}{21}$

3 SUBTRACT

$\frac{12}{21}$ - $\frac{7}{21}$ = $\frac{5}{21}$

5/21 = 0.23809...

4 DIVIDE BY 0.4

0.23809... / 0.4 = 0.5952...

5 ROUND TO 2 D.P.

0.5952... -> 3rd d.p. is 5 -> round up

ANSWER: 0.60

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Questions 49 to 52 - Arrange in Order of Size (Smallest First)

TIP: To compare fractions and decimals: convert everything to decimals first (divide top by bottom), then arrange from smallest to largest.

Q49. Arrange in order: 1/3, 0.33, 4/15

1 CONVERT EVERYTHING TO DECIMALS

1/3 = 0.3333 0.33 = 0.33 4/15 = 0.2667

2 ARRANGE SMALLEST TO LARGEST

0.2666666666666666 (4/15) < 0.33 (0.33) < 0.3333333333333333 (1/3)

ANSWER: 4/15 < 0.33 < 1/3

Q50. Arrange in order: 2/7, 0.3, 4/9

1 CONVERT EVERYTHING TO DECIMALS

2/7 = 0.2857 0.3 = 0.3 4/9 = 0.4444

2 ARRANGE SMALLEST TO LARGEST

0.2857142857142857 (2/7) < 0.3 (0.3) < 0.4444444444444444 (4/9)

ANSWER: 2/7 < 0.3 < 4/9

Q51. Arrange in order: 0.71, 7/11, 0.705

1 CONVERT EVERYTHING TO DECIMALS

0.71 = 0.71 7/11 = 0.6363 0.705 = 0.705

2 ARRANGE SMALLEST TO LARGEST

0.6363636363636364 (7/11) < 0.705 (0.705) < 0.71 (0.71)

ANSWER: 7/11 < 0.705 < 0.71

Q52. Arrange in order: 4/13, 0.3, 5/18

1 CONVERT EVERYTHING TO DECIMALS

4/13 = 0.3077 0.3 = 0.3 5/18 = 0.2778

2 ARRANGE SMALLEST TO LARGEST

0.2777777777777778 (5/18) < 0.3 (0.3) < 0.3076923076923077 (4/13)

ANSWER: 5/18 < 0.3 < 4/13

Questions 53 to 60 - Convert Recurring Decimals to Fractions

TIP: Algebraic method: Let x = the recurring decimal. Multiply by the right power of 10 to shift the recurring block. Subtract the original equation. Solve for x. Simplify.

Q53. Convert 0.6 (recurring) to a fraction

1. **LET X EQUAL THE RECURRING DECIMAL**
   x = 0.66... (the 6 part repeats)
2. **1 DIGIT(S) RECUR -> MULTIPLY BY 10**
   10x = 6.6...
   Multiplying by 10 shifts 1 decimal place(s)
3. **SUBTRACT ORIGINAL EQUATION**
   10x - x = 6.6... - 0.6... -> 9x = 6
   The recurring part cancels out
4. **SOLVE AND SIMPLIFY**
   x = 6 / 9 = 2/3 (divide both by GCF 3)

ANSWER: $\frac{2}{3}$

Q54. Convert 0.4 (recurring) to a fraction

1. **LET X EQUAL THE RECURRING DECIMAL**
   x = 0.44... (the 4 part repeats)
2. **1 DIGIT(S) RECUR -> MULTIPLY BY 10**
   10x = 4.4...
   Multiplying by 10 shifts 1 decimal place(s)
3. **SUBTRACT ORIGINAL EQUATION**
   10x - x = 4.4... - 0.4... -> 9x = 4
   The recurring part cancels out
4. **SOLVE AND SIMPLIFY**
   x = 4 / 9

ANSWER: $\frac{4}{9}$

Q55. Convert 0.12 (recurring) to a fraction

1 **LET X EQUAL THE RECURRING DECIMAL**

x = 0.1212... (the 12 part repeats)

2 **2 DIGIT(S) RECUR -> MULTIPLY BY 100**

100x = 12.12...

Multiplying by 100 shifts 2 decimal place(s)

3 **SUBTRACT ORIGINAL EQUATION**

100x - x = 12.12... - 0.12... -> 99x = 12

The recurring part cancels out

4 **SOLVE AND SIMPLIFY**

x = 12 / 99 = 4/33 (divide both by GCF 3)

ANSWER: $\frac{4}{33}$

Q56. Convert 0.43 (recurring) to a fraction

1 **LET X EQUAL THE RECURRING DECIMAL**

x = 0.4343... (the 43 part repeats)

2 **2 DIGIT(S) RECUR -> MULTIPLY BY 100**

100x = 43.43...

Multiplying by 100 shifts 2 decimal place(s)

3 **SUBTRACT ORIGINAL EQUATION**

100x - x = 43.43... - 0.43... -> 99x = 43

The recurring part cancels out

4 **SOLVE AND SIMPLIFY**

x = 43 / 99

ANSWER: $\frac{43}{99}$

Q57. Convert 0.134 (recurring) to a fraction

1 **LET X EQUAL THE RECURRING DECIMAL**

x = 0.134134... (the 134 part repeats)

2 **3 DIGIT(S) RECUR -> MULTIPLY BY 1000**

1000x = 134.134...

Multiplying by 1000 shifts 3 decimal place(s)

3 **SUBTRACT ORIGINAL EQUATION**

1000x - x = 134.134... - 0.134... -> 999x = 134

The recurring part cancels out

4 SOLVE AND SIMPLIFY

x = 134 / 999

ANSWER: $\frac{134}{999}$

Q58. Convert 0.731 (recurring) to a fraction

1 LET X EQUAL THE RECURRING DECIMAL

x = 0.731731... (the 731 part repeats)

2 3 DIGIT(S) RECUR -> MULTIPLY BY 1000

1000x = 731.731...

Multiplying by 1000 shifts 3 decimal place(s)

3 SUBTRACT ORIGINAL EQUATION

1000x - x = 731.731... - 0.731... -> 999x = 731

The recurring part cancels out

4 SOLVE AND SIMPLIFY

x = 731 / 999

ANSWER: $\frac{731}{999}$

Q59. Convert 0.25 (recurring) to a fraction

1 LET X EQUAL THE RECURRING DECIMAL

x = 0.255... (the 5 part repeats)

2 MULTIPLY BY 10 TO POSITION RECURRING AFTER DECIMAL

10x = 2.5...

3 MULTIPLY BY 100 FOR ONE FULL CYCLE

100x = 25.5...

4 SUBTRACT: 100X - 10X

90x = 23

Recurring part cancels

5 SOLVE AND SIMPLIFY

x = 23 / 90

ANSWER: $\frac{23}{90}$

Q60. Convert 0.617 (recurring) to a fraction

1 LET X EQUAL THE RECURRING DECIMAL

x = 0.61717... (the 17 part repeats)

2 MULTIPLY BY 10 TO POSITION RECURRING AFTER DECIMAL

10x = 6.17...

3 MULTIPLY BY 1000 FOR ONE FULL CYCLE

1000x = 617.17...

4 SUBTRACT: 1000X - 10X

990x = 611

Recurring part cancels

5 SOLVE AND SIMPLIFY

x = 611 / 990

ANSWER: $\frac{611}{990}$

Exercise 1.1D

Fractions and Decimals - Full Step-by-Step Solutions

Question 61 - Decide Whether Each Statement is True or False

Q61a. 0.7 < 7/9

1	Convert 7/9 to decimal: 7 / 9 = 0.777... = 0.7 recurring
2	0.7 vs 0.777... -> 0.7 is LESS THAN 0.777...
3	0.7 < 7/9 is TRUE

VERDICT: TRUE

Q61b. 82% > 0.823 (recurring)

1	Convert 82% to decimal: 82 / 100 = 0.82
2	0.823 recurring = 0.8232323...
3	0.82 vs 0.8232... -> 0.82 is LESS THAN 0.8232...
4	82% > 0.823 recurring is FALSE (it is actually less)

VERDICT: FALSE

Q61c. 0.63 (recurring) ≠ 7/11

1	Convert 7/11 to decimal: 7 / 11 = 0.636363... = 0.63 recurring
2	0.63 recurring = 0.6363... = 7/11
3	They ARE equal, so 0.63 recurring ≠ 7/11 is FALSE

VERDICT: FALSE